Screening of electrostatic potential in a composite fermion system

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Screening of the electric field of a test charge by monolayer and double-layer composite fermion systems is considered. It is shown that the electric field of the test charge is partly screened at distances much larger than the magnetic length. The value of screening as a function of the distance depends considerably on the filling factor. The effect of variation of the value of screening in the double-layer system upon a transition to a state described by the Halperin wave function is determined.

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The model of composite fermions was proposed by Jain [1] to describe the fractional filling factor hierarchy observed in quantum Hall systems. It was shown in Ref. [1] that the Laughlin wave function has a topological structure equivalent to that of a system of quasiparticles carrying the statistical charge and the statistical gauge field flux. In the mean field approximation, the interaction of composite quasiparticles with the gauge field is reduced to the action of a self-consistent field which partially screens the external magnetic field. Consequently, the fractional quantum Hall effect in the electron system emerges as an integer quantum Hall effect in a system of composite fermions.

Lopez and Fradkin [2] and Halperin, Lee, and Read [3] developed the Chern–Simons formalism for describing a system of composite fermions. The formalism allows us to introduce systematically the corrections to the mean-field solution by expanding the effective Lagrangian in small deviations of the gauge field from the mean-field configuration.

The approach developed in Refs. [2,3] was generalized by Lopez and Fradkin [4] to the case of a double-layer system. A specific feature of such a system is the possibility of formation of generalized Laughlin states whose multiparticle wave function is characterized by an additional set of zeros for coinciding x, y coordinates of electrons in the opposite layers. The wave function for such states was proposed by Halperin. [5] Although the original analysis presented in Ref. [5] concerns a monolayer system of unpolarized electrons, the wave function proposed by Halperin is generalized to the case of a two-layered system, if pseudospins, which corresponds to the layer index are introduced. The states [5] may appear for new (different from monolayer) filling factors. In particular, the quantum Hall effect emerges for a filling factor $\nu = 1/2$, which is indeed observed in experiments [6,7]. Moreover, for certain fixed values of the filling factor in two-layered systems, a phase transition between different generalized Laughlin states becomes possible upon a change in the separation between the layers (see, for instance, [8])

In this work, we study the effect of screening of the external electrostatic potential by a composite fermion system. We consider the screening of the field of a test charge located at the boundary of a semi-infinite medium with the dielectric constant ϵ, having at a certain distance from the boundary a two-dimensional electron layer (or a double layer system). We found that for incompressible fractional quantum Hall states the electric field E of the test charge may deviate considerably from unscreened field at distances much larger than the magnetic length. The specific form of the dependence $E(r)$ is defined by the ground state of the electron system.

I. A MONOLAYER SYSTEM IN AN INFINITE MEDIUM

To begin with, let us consider the problem of screening in an infinite medium containing a two-dimensional electron layer in the fractional quantum Hall regime. In order to describe the system, we consider the model of spinless fermions Ψ (it is assumed that electrons are completely polarized in spin) interacting with the two-dimensional Chern-Simons gauge field a_μ and the electromagnetic field A_μ. The action of the system has the form

$$S = S_{\rm CF} + S_{\rm em}, \tag{1}$$

where

$$S_{\rm CF} = \int dt d^2 r [\Psi^*(\mathbf{r})(i\partial_t + \mu - a_0 - eA_0 \\ -\frac{1}{2m}(i\nabla_2 + \mathbf{a} + \frac{e}{c}\mathbf{A}^{\rm pl})^2)\Psi(\mathbf{r}) + \frac{1}{2\pi\varphi} a_0 b] \ , \tag{2}$$

$$S_{\rm em} = \frac{1}{8\pi}\int dt d^3 r (\epsilon \mathbf{E}^2 - \mathbf{B}^2). \tag{3}$$

In Eq. (2), m is the mass of composite fermions, μ the chemical potential, $b = \partial_x a_y - \partial_y a_x$, the "magnetic" component of the gauge field, φ, the number of gauge field flux quanta carried by a composite quasiparticle (φ is even). It is assumed that the distribution of composite fermions along the z-axis is described by *delta*-function.

The transverse gauge is used for the field a ($\partial_i a_i = 0$). For the electromagnetic field also, we used the transverse gauge in the plane ($\partial_x A_x + \partial_y A_y = 0$).

Functional integration with respect to the field Ψ in the expression for the partition function of the system

$$Z = \int D\Psi^* D\Psi Da_\mu \exp(iS) \quad (4)$$

gives the following effective action for a system of interacting gauge and electromagnetic fields:

$$S_{\rm eff}(a, A) = -i \ {\rm Tr} \ \log\left[i\partial_t + \mu - a_0 - eA_0 - \frac{1}{2m}(i\nabla_2 + \mathbf{a} + \frac{e}{c}\mathbf{A}^{\rm pl})^2\right] + \int dt d^2 r \frac{1}{2\pi\varphi} a_0 b + S_{\rm em}. \quad (5)$$

The condition of stationary configuration of the action (5) upon variation of the field a_μ defines the value of the self-consistent effective field acting on composite fermions:

$$B_{\rm eff} = B - \frac{2\pi c\varphi}{|e|} n_0, \quad (6)$$

where n_0 is the uniform electron concentration. The fractional quantum Hall effect is observed for an integral number N of filled Landau levels in a field $B_{\rm eff}$, which corresponds to the filling factor $\nu = N/(\varphi N + {\rm sign} B_{\rm eff})$.

We shall confine the subsequent analysis to a consideration of the part of the effective action (5) that is quadratic in fluctuations of the field a_μ and A_μ. In order to solve the problem considered here, we consider only static fluctuations. Expansion of the action (5) in the vicinity of the stationary configuration gives

$$S^{(2)}_{\rm eff}(a, A) = \frac{1}{2}\int dt d^2 q [(a^*_{\mu\mathbf{q}} + \tilde{A}^*_{\mu\mathbf{q}})\Pi^{\Psi}_{\mu\nu q}(a_{\nu\mathbf{q}} + \tilde{A}_{\nu\mathbf{q}}) + a^*_{\mu\mathbf{q}}\Pi^{\rm CS}_{\mu\nu q} a_{\nu\mathbf{q}}] + \frac{1}{2}\int dt d^2 q dq_z A^*_{\mu\mathbf{q}q_z}\Pi^{\rm em}_{\mu\nu q q_z} A_{\nu\mathbf{q}q_z}, \quad (7)$$

where the subscripts μ and ν assume the values 0 and 1 corresponding respectively to zeroth and transverse components of the fields a_μ and A_μ (in Eq. (7), we disregard the contribution of the component A_z which can be put equal to zero without any loss of generality in the static problem under consideration), and $\mathbf{q}$ is the wave vector component parallel to the (x, y) plane. In Eq. (7), $\tilde{A}_{0\mathbf{q}} = eA_{0\mathbf{q}}(z = 0)$, $\tilde{A}_{1\mathbf{q}} = (e/c)A_{1\mathbf{q}}(z = 0)$,

$$\Pi^{\Psi}_{\mu\nu q} = -\frac{1}{2\pi\omega_{\rm c}}\begin{pmatrix} q^2\Sigma_0 & iq\omega_{\rm c}\Sigma_1 \\ -iq\omega_{\rm c}\Sigma_1 & \omega^2_{\rm c}(\Sigma_2 + N)\end{pmatrix}, \quad (8)$$

$$\Pi^{\rm CS}_{\mu\nu q} = \frac{1}{2\pi\varphi}\begin{pmatrix} 0 & iq \\ -iq & 0\end{pmatrix}, \quad (9)$$

$$\Pi^{\rm em}_{\mu\nu q q_z} = \frac{1}{4\pi}\begin{pmatrix} \epsilon(q^2 + q_z^2) & 0 \\ 0 & -(q^2 + q_z^2)\end{pmatrix}. \quad (10)$$

In Eq. (8), we have introduced the notation

$$\Sigma_j = -({\rm sign} B_{\rm eff})^j e^{-x}\sum_{n=0}^{N-1}\sum_{m=N}^{\infty}\frac{n!}{m!}\frac{x^{m-n-1}}{(m-n)} \times [L_n^{m-n}(x)]^{2-j} \times \left((m - n - x)L_n^{m-n}(x) + 2x\frac{dL_n^{m-n}(x)}{dx}\right)^j, \quad (11)$$

where $x = (ql_{\rm eff})^2/2$, $l_{\rm eff} = (N/2\pi n_0)^{1/2}$ is the effective magnetic length, $\omega_{\rm c} = 2\pi n_0/(mN)$ is the effective cyclotron frequency, and $L_n^{m-n}(x)$ is the generalized Laguerre polynomial. The quantities (11) are calculated through the Green's current–current functions for the fermion system in the field $B_{\rm eff}$ (the temperature is assumed to be equal to zero). Expressions of the type (11) were first derived in the theory of anyons (see, for instance, [9]).

Integration over fluctuations of the field a leads to the following expression for the action of the electromagnetic field:

$$S(A) = \frac{1}{4\pi}\int dt dq_z dq'_z d^2 q A^*_{\mu\mathbf{q}q_z}\Lambda_{\mu\nu q}A_{\nu\mathbf{q}q'_z} + \frac{1}{2}\int dt dq_z d^2 q A^*_{\mu\mathbf{q}q_z}\Pi^{\rm em}_{\mu\nu q q_z}A_{\nu\mathbf{q}q_z}, \quad (12)$$

where

$$\Lambda_{\mu\nu q} = -\frac{e^2}{2\pi\omega_{\rm c}\Delta_1}\begin{pmatrix} q^2\Sigma_0 & iq\omega_{\rm c}D/c \\ -iq\omega_{\rm c}D/c & \omega^2_{\rm c}(\Sigma_2 + N)/c^2\end{pmatrix} \quad (13)$$

with

$$D = \Sigma_1 + \varphi(\Sigma_0(\Sigma_2 + N) - \Sigma_1^2), \quad (14)$$

$$\Delta_1 = (1 - \varphi\Sigma_1)^2 - \varphi^2\Sigma_0(\Sigma_2 + N). \quad (15)$$

The action (12) leads to the following expression for the electromagnetic field potential in a system with a test charge $e_{\rm ext}$ placed at the origin:

$$\frac{1}{2\pi}\int dq'_z\Lambda_{\mu\nu q}A_{\nu\mathbf{q}q'_z} + \Pi^{\rm em}_{\mu\nu q q_z}A_{\nu\mathbf{q}q_z} = \delta_{\mu 0} j_0, \quad (16)$$

where $j_0 = e_{\rm ext}/(2\pi)^{3/2}$. The solution of Eq. (16) is sought in the form

$$A_{\mu\mathbf{q}q_z} = \frac{C_\mu(q)}{q^2 + q_z^2}. \quad (17)$$

Consequently, we obtain the following expression for the quantity $A_{0\mathbf{q}q_z}$:

$$A_{0\mathbf{q}q_z} = \frac{4\pi j_0}{q^2 + q_z^2} \times \left[\epsilon - \frac{qe^4c^{-2}\omega_{\rm c}(\Sigma_0(\Sigma_2 + N) - \Sigma_1^2) + q^2e^2\Sigma_0}{e^2c^{-2}\omega^2_{\rm c}(\Sigma_2 + N) + q\omega_{\rm c}\Delta_1}\right]^{-1} \quad (18)$$

In the electrostatic limit ($c \to \infty$), Eq. (18) is reduced to the form

$$A_{0\mathbf{q}q_z} = \frac{4\pi j_0}{\epsilon}\frac{1}{q^2+q_z^2}\left(1+\frac{f_q\Sigma_0}{\Delta_1 - f_q\Sigma_0}\right), \quad (19)$$

where $f_q = e^2q/\epsilon\omega_c$. Eq. (19) differs from Eq. (18) significantly only for

$$q < \frac{e^2\omega_c}{c^2}. \quad (20)$$

The values of q satisfying this inequality are several orders of magnitude lower than the characteristic scale of wave vectors of the problem $\sim l_{\text{eff}}^{-1}$. A consideration of the difference between Eqs. (18) and (19) in the region (20) leads to a very weak screening of the electric field of the test charge at large distances. Here and below, we shall not analyze this very weak effect, but confine to the approximate expression (19). Note that this approximation corresponds to negligible nondiagonal components of the tensor Λ in Eq. (16), which will be taken into consideration in the following sections.

The expression for the screened electric field of a test charge, calculated from Eq. (19) for $z = 0$, has the form

$$E_{\text{pl}}(r) = -\frac{e_{\text{ext}}}{\epsilon r^2}(1+F(r)), \quad (21)$$

where r is the distance up to the test charge, and

$$F(r) = r^2\int dq J_1(qr)\frac{qf_q\Sigma_0}{\Delta_1 - f_q\Sigma_0} \quad (22)$$

($J_i(x)$ is the Bessel function).

To complete the picture, we also present the expression for the z-component of the magnetic field (for $z = 0$), induced by the test charge:

$$\delta B_z(r) = -\frac{e_{\text{ext}}e^2}{c\epsilon}\int dqqJ_0(qr)\frac{D}{\Delta_1 - f_q\Sigma_0}. \quad (23)$$

In this expression also, we have omitted the correction emerging in the region (20). For characteristic values of the parameters and for $e_{\text{ext}} = e$, the numerical estimate for δB_z is found to be smaller than 1 Oe for all r, i.e., the effect is of theoretical interest only. From the experimental point of view, the most important effect is associated with the deviation of $F(r)$ from zero in Eq. (21), which may be of the order of unity for finite values of r. The specific form of the dependence $F(r)$ is determined by the ground state of the quantum Hall system and is modified significantly upon a transition to another Hall step. The following two sections are devoted to an analysis of this effect in a semi-infinite medium.

II. A MONOLAYER SYSTEM IN A SEMI-INFINITE MEDIUM

Let us now consider the geometry which seems to be appropriate for observing the screening effect. We shall assume that the test charge is located at the surface of a semi-infinite medium with the dielectric constant ϵ. At a distance a from the surface, the medium contains a two-dimensional electron layer in the fractional quantum Hall regime.We shall seek an expression for the screened field of a test charge at the interface.

Disregarding the nondiagonal components of the tensor Λ, we can present Eq. (16) in the geometry under consideration as follows:

$$\frac{1}{2\pi}\int dq_z'(e^{i(q_z-q_z')a}\Lambda_{00q} + \epsilon_{q_z-q_z'}\frac{q^2+q_zq_z'}{2})A_{0\mathbf{q}q_z'} = j_0, \quad (24)$$

where

$$\epsilon_{q_z} = \frac{\epsilon+1}{2}\delta(q_z) + i\frac{\epsilon-1}{2\pi}P(\frac{1}{q_z}). \quad (25)$$

The solution of Eq. (24) is sought in the form

$$A_{0\mathbf{q}q_z} = \frac{C_1(q)+C_2(q)e^{iq_za}}{q^2+q_z^2}. \quad (26)$$

As a result, we arrive at the following expression for the electric field projection on the plane (x, y) at the interface:

$$E_{\text{pl}}(r) = -\frac{e_{\text{ext}}}{\epsilon' r^2}(1+F(r)), \quad (27)$$

where

$$F(r) = \frac{\epsilon}{\epsilon'}r^2\int dqJ_1(qr)\frac{qf_q\Sigma_0e^{-2qa}}{\Delta_1 - f_q\Sigma_0(1+\frac{\epsilon-1}{\epsilon+1}e^{-2qa})} \quad (28)$$

with $\epsilon' = (\epsilon+1)/2$.

The dependence $F(r)$ is shown in Fig.1 for the filling factors $\nu = 1/3,\ 3/7,\ 5/11$. Calculations were made by using the following values of the parameters: $n_0 = 10^{11}$ cm^{-2}, $m = 0.25m_e$, $\epsilon = 12,6$, $a = 500\mathring{A}$. It can be seen from Fig 1 that the system has a significant screening of the electric field of the test charge at distances considerably larger than the magnetic length. As the filling factor ν approaches the value 1/2, the dependence $F(r)$ becomes oscillatory. Note that an increase in the value of a weakens the effect. A decrease in this parameter leads not only to an increase in the amplitude of the effect, but also to a wider range of filling factors for which $F(r)$ oscillates as a function of the distance. In particular, numerical computations for the geometry considered in Sec. I lead to an oscillatory dependence $F(r)$ for all filling factors corresponding to a fractional quantum Hall effect. In order to observe these oscillations, one can measure the electric field inside the dielectric medium. For the case considered in this section, oscillations emerge when the effective magnetic length becomes of the order of $2a$.

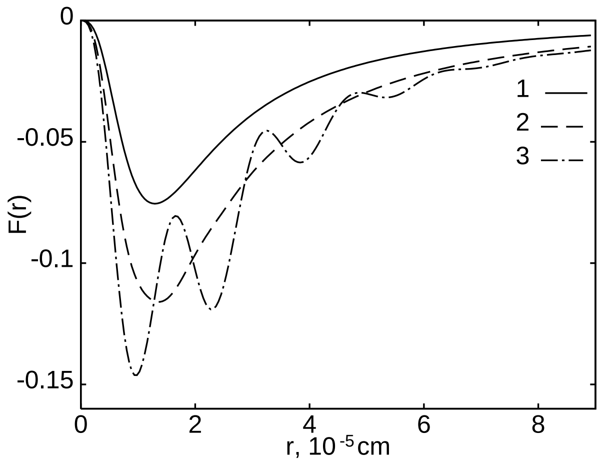

FIG. 1. Relative screening of the test charge field by the monolayer system. 1 - $\nu = 1/3$; 2 - $\nu = 3/7$; 3 - $\nu = 5/11$.

III. A DOUBLE-LAYER SYSTEM IN A SEMI-INFINITE MEDIUM

Let us now consider the screening of the field of a test charge by a double-layer electron system. We shall consider a system in which the states described by the Halperin's wave function [5] are realized. Such a system can be described by introducing two types of Chern–Simons fields corresponding to statistical charges belonging to composite quasiparticles in opposite layers, and to an additional term in the Lagrangian of the system, which is nondiagonal in the gauge fields. The action of the system has the form

$$S_{\rm CF} = \int dt d^2 r \sum_{k=1}^{2} [\Psi_k^*(\mathbf{r})(i\partial_t + \mu - a_{k0} - eA_{k0} - \frac{1}{2m}(i\nabla_2 + \mathbf{a}_k + \frac{e}{c}\mathbf{A}_k^{\rm pl})^2)\Psi_k(\mathbf{r}) + \sum_{k,k'=1}^{2} \Theta_{kk'} a_{k0} b_{k'}], \quad (29)$$

where A_{k0} and $\mathbf{A}_k^{\rm pl}$ are the scalar and vector components of the electromagnetic field potential in the layer k, and

$$\Theta_{kk'} = \frac{1}{2\pi(\varphi^2 - s^2)} \begin{pmatrix} \varphi & -s \\ -s & \varphi \end{pmatrix}. \quad (30)$$

In this equation, φ and s are the numbers of the gauge field flux quanta carried by a composite quasiparticle, which correspond to the statistical charges of quasiparticles in the same layer and in the opposite layer respectively (φ is even while s is an arbitrary integer). For the sake of simplicity, we confine the analysis to two equivalent layers. Fractional quantum Hall effect in the system (29) is realized for filling factors $\nu = 2N/((\varphi+s)N + {\rm sign}B_{\rm eff})$ ($\nu = 2\nu_i$, where ν_i is the filling factor per layer). Carrying out a procedure analogous to the one carried out in Sec. I, we arrive at the following expression for the action of the electromagnetic field of the system (29):

$$S(A) = \frac{1}{2}\int dt d^2 q A^*_{k\mu\mathbf{q}} \Lambda_{kk'\mu\nu q} A_{k'\nu\mathbf{q}} + S_{\rm em}, \quad (31)$$

where

$$\hat{\Lambda}_{kk'} = \frac{1}{2}\begin{pmatrix} \hat{\Lambda}^+ + \hat{\Lambda}^- & \hat{\Lambda}^+ - \hat{\Lambda}^- \\ \hat{\Lambda}^+ - \hat{\Lambda}^- & \hat{\Lambda}^+ + \hat{\Lambda}^- \end{pmatrix}. \quad (32)$$

Matrices $\hat{\Lambda}^+$ and $\hat{\Lambda}^-$ in Eq. (32) are defined by Eqs. (13)–(15) in which the parameter φ is replaced by $\varphi + s$ and $\varphi - s$ respectively. Note that although Eqs. (29) and (30) become meaningless for $\varphi = s$, the expressions (31) and (32) remain valid even in this case. The situation $\varphi = s$ corresponds to infinite rigidity of the antiphase oscillations of the gauge fields. Going over in Eqs. (29) and (30) to new variables corresponding to synphase and antiphase oscillations, we must consider for the integration variables in the case $\varphi = s$ only synphase oscillations of the fields $a_{i\mu}$ and put the variable corresponding to antiphase oscillations equal to zero. It can be verified that in this case also we arrive at the relations (31) and (32).

For the geometry in which the z-coordinate of the first and second layer is equal to $-a$ and $-(a+d)$ respectively, we obtain from Eq. (31) the following equation for the scalar potential of the system:

$$\frac{1}{4\pi}\int dq'_z \{ e^{i(q_z - q'_z)a}[(\Lambda^+_{00q} + \Lambda^-_{00q})(1 + e^{i(q_z - q'_z)d}) + (\Lambda^+_{00q} - \Lambda^-_{00q})(e^{-iq'_z d} + e^{iq_z d})] + \epsilon_{q_z - q'_z}(q^2 + q_z q'_z)\} A_{0\mathbf{q}q'_z} = j_0, \quad (33)$$

where the nondiagonal components of the tensors Λ^+ and Λ^- are neglected as before.

The solution of Eq. (33) is sought in the form

$$A_{0\mathbf{q}q_z} = \frac{C_1(q) + C_2(q)e^{iq_z a} + C_3(q)e^{iq_z(a+d)}}{q^2 + q_z^2}. \quad (34)$$

As a result, we arrive at an expression for the electric field projection on the plane (x, y) for $z = 0$, whose form coincides with Eq. (27) in which the function $F(r)$ is modified to the form

$$F(r) = \frac{\epsilon}{\epsilon'} r^2 \int dq J_1(qr) \frac{q S_q e^{-2qa}}{R_q - \frac{\epsilon-1}{\epsilon+1} S_q e^{-2qa})}, \quad (35)$$

where

$$R_q = (\Delta_1^+ - f_q E_q^+ \Sigma_0)(\Delta_1^- - f_q E_q^- \Sigma_0), \quad (36)$$

$$S_q = f_q \Sigma_0 [\frac{1}{2}(E_q^+)^2 \Delta_1^- + \frac{1}{2}(E_q^-)^2 \Delta_1^+ - f_q E_q^+ E_q^- \Sigma_0]. \quad (37)$$

In formulas (36) and (37), the functions Δ_1^+ and Δ_1^- are defined by formula (13) in which φ is replaced by $\varphi + s$ and $\varphi - s$ respectively, and $E_q^{\pm} = (1 \pm e^{-qd})$.

It can be seen from Eqs. (35)–(37) that screening in a two-layer system depends on parameter $\varphi + s$ as well as $\varphi - s$. Hence for a fixed value of the filling factor which depends only on the first of these parameters, the expression (35) differs for the cases $s = 0$ and $s \neq 0$. Consequently, a transition from a state with $s = 0$ to a state with nonzero s (such a state corresponds to the Halperin's wave function) will be manifested in a variation of the dependence of screened field of a test charge on the distance.

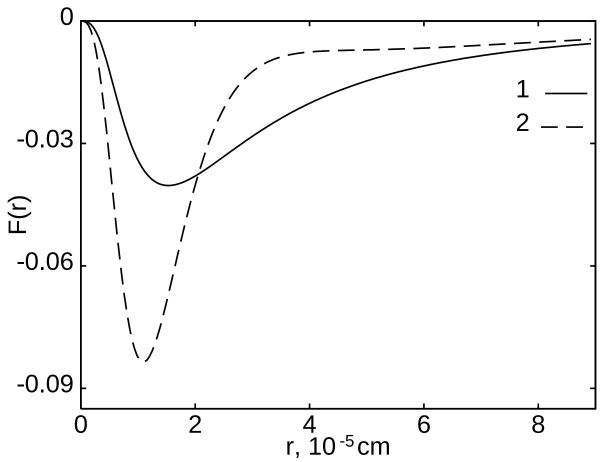

FIG. 2. Relative screening of the test charge field by the double-layer system for $\nu = 2/5$. 1 - $\varphi = 4$, $s = 0$; 2 - $\varphi = 2$, $s = 2$.

Let us consider a system with $\nu = 2/5$. In such a system, the fractional quantum Hall effect may correspond to sets of parameters ($\varphi = 4$ and $s = 0$) and ($\varphi = 2$ and $s = 2$). Fig. 2 shows the dependence $F(r)$ for these two cases for the same values of the system parameters as in Sec. II and for $d = 400$ Å. It can be seen from the curves that the dependences $F(r)$ differ considerably for these two cases. An experimental observation of such a sharp variation in screening upon a slight variation in the separation between layers points towards a phase transition between different ground states in a two-layer system. Note that the situation considered in this work differs from the one considered by us in Ref. [10] where we studied screening in a two-layer system in an infinite medium with two test charges located in the opposite layers. In such a case, the screening of the test charge field depends only on parameter $\varphi + s$, and the variation of the spatial distribution of the induced charge during a phase transition between generalized Laughlin states is associated just with a variation of this parameter with a simultaneous reversal of the sign of B_{eff}. Only a few of the possible transitions satisfy this condition. In particular, the transition considered above for $\nu = 2/5$ (which is most suitable for observation since it corresponds to lowest level in the hierarchy of the generalized Laughlin states) does not satisfy such a condition. The origin of the effect considered in this work is associated with the asymmetric arrangement of the test charge relative to the two-layer system. In particular, this is manifested in that a decrease in the separation between layers leads to a suppression of the effect.

Thus, we have considered in this work the screening of the electric field of a test charge by a monolayer and a two-layer systems of composite fermions. The expressions for the screened field are obtained by taking into account the effect of the interface. It is shown that a partial screening of the test charge electric field occurs in the system at distances much larger than the magnetic length. The specific form of the dependence of the screened field on the distance from the test charge is modified considerably upon a variation of the ground state of the system. The observation of the screening effect as a function of the filling factor and separation between layers (in a two-layer system) can be treated as a possible experimental verification of the model of composite fermions and the method of observing changes in the topological order in fractional quantum Hall systems. The solutions obtained in this work pertain to the case when the test charge and the electric field detector are placed on the surface of the sample containing a two-dimensional electron layer. The approach used in this work can be modified to describe a different geometry of the experiment.

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